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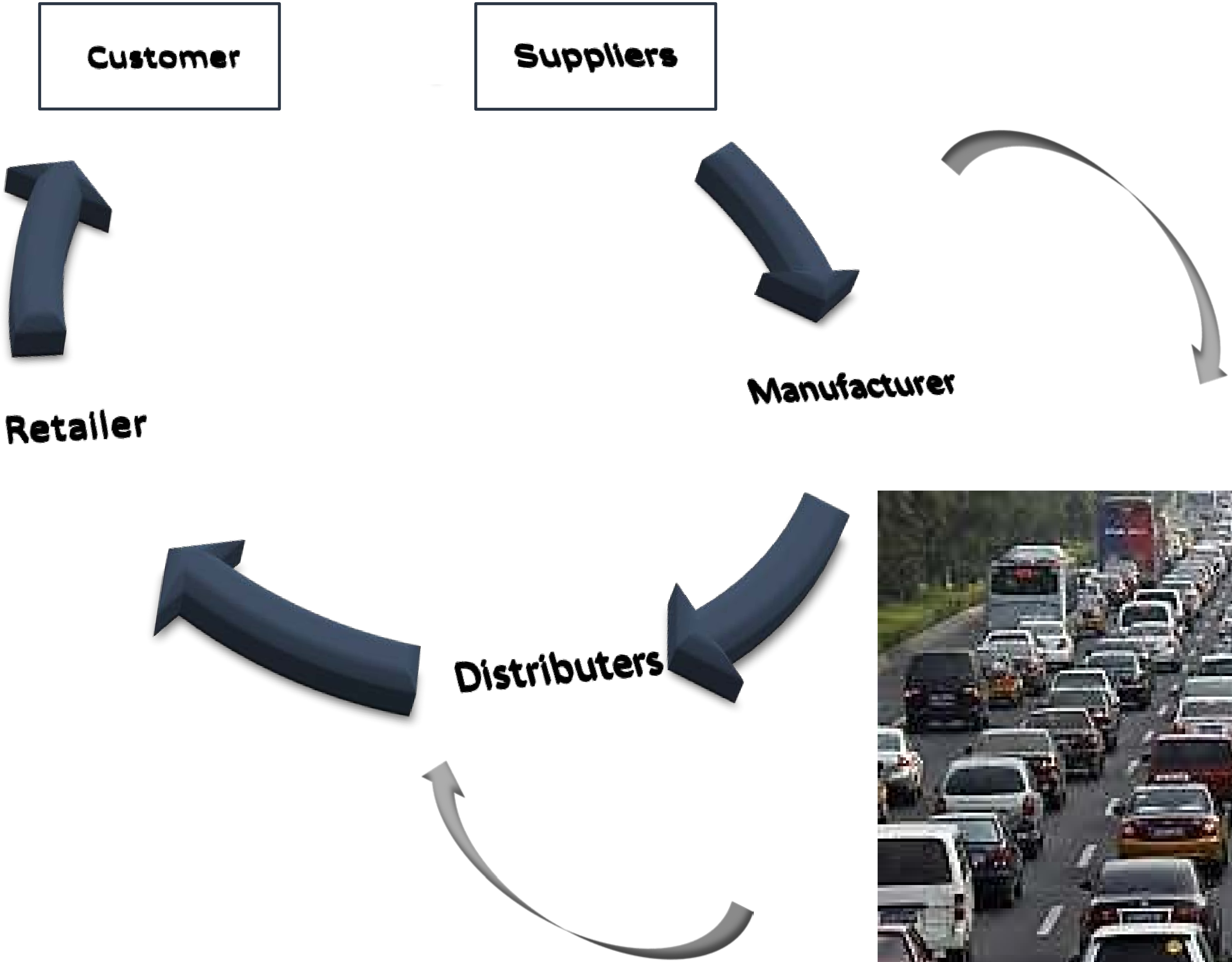
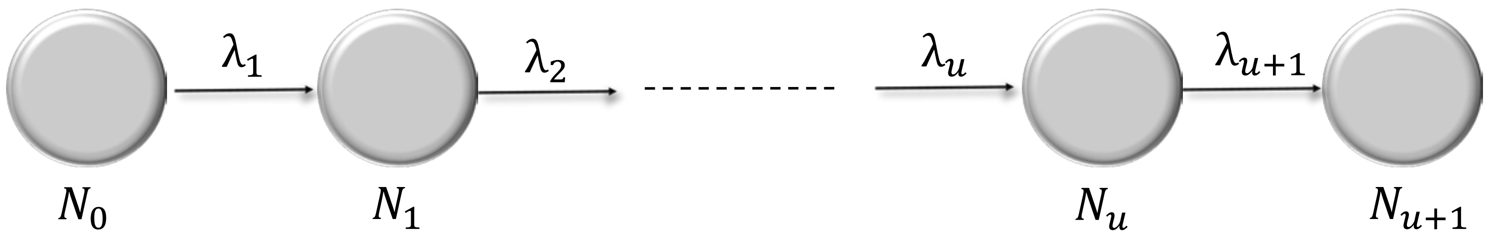
Modelling of supply chains inspired by traffic dynamics


Customer
Suppliers
Manufacturer
Retailer
Distributers

One-dimensional supply-chain



- Suppliers N_b b ϵ $\{0, \dots, u\}$
- Consumers N_{u+1}
- Feeding rate λ_b b ϵ $\{0, \dots, u+1\}$

- Dynamics inventory :

$$\frac{dN_b(t)}{dt} = \lambda_b(t) - \lambda_{b+1}(t)$$

$\lambda_b(t)$ ↓ **Rate received products**

$\lambda_{b+1}(t)$ ↓ **Rate delivered products**

- Dynamics delivery rate :

$$\frac{d\lambda_b(t)}{dt} = \frac{\left(W_b(t) - \lambda_b(t)\right)}{\tau}$$

τ = adaptation time interval

$W_b(t)$ = desired delivery rate

Management function

$W_b(t) = W_b\,(\{N_a(t)\}\,,\{N_a(t+\ \Delta t)\})$ where $N_a(t+\ \Delta t) \approx N_a\ (t) +\ \ \Delta t\,\frac{dN_a(t)}{dt}$ ⟶ General strategy

Our strategy ⟶ $$W_b(t) = W\left(N_{(b)}(t)\right)$$

$$N_{(b)}(t) = \sum_{c=-n}^{n} w_c(N_{b+c}(t) +\ \ \Delta t\,\frac{dN_{b+c}(t)}{dt})$$ ⟶ Weighted mean value

w_c= 0 if $b + c < 0\ \ or\ \ b + c > u$

$$\sum_{c=-n}^{n} w_c = 1$$ ⟶ Normalization condition

$\Delta t =$ forecast time horizon

Stability condition

Dynamic equations in the vicinity of the stationary state :

Stationary solutions

- Inventory : N_0
- Delivery rate : $\lambda_0 = W(N_0)$

$$\delta N_b(t) = N_b(t) - N_0$$

$$\delta\lambda_b(t) = \lambda_b(t) - W(N_0)$$

$$\tau < \Delta t + \frac{1}{|W'(N_0)|}\left(\frac{1}{2} + \sum_{c=-n}^{n} c w_c\right)$$

Dynamical solution in the vicinity of the stationary state (I)

Deviation of the inventory:

$$\delta N_b(t) = N_b(t) - N_0$$

$$\frac{d\delta N_b(t)}{dt} = \delta\lambda_b(t) - \delta\lambda_{b+1}(t)$$

Deviation of the delivery rate:

$$\delta\lambda_b(t) = \lambda_b(t) - W(N_0)$$

$$\frac{d\delta\lambda_b(t)}{dt} = \frac{1}{\tau}\left[W'(N_0)\left(\delta N_b(t) + \Delta t\,\frac{d\delta N_b(t)}{dt}\right) - \delta\lambda_b(t)\right]$$

Deriving and substituting…

Dynamical solution in the vicinity of the stationary state (II)

Damped forced harmonic oscillator model

... we obtain: $$\boldsymbol{\delta\ddot{\lambda}_b(t) + 2\gamma\delta\dot{\lambda}_b(t) + {w_0}^2\delta\lambda_b(t) = f_b(t)}$$

$$2\gamma = \frac{1 + |W'(N_0)|\Delta t}{\tau} \qquad {w_0}^2 = \frac{|W'(N_0)|}{\tau} \qquad f_b(t) = \frac{|W'(N_0)|}{\tau}\left(\delta\lambda_{b+1}(t) + \Delta t\frac{d\delta\lambda_{b+1}(t)}{dt}\right)$$

Solution at steady state under the effect of a periodic oscillation of the form → $f_u(t) = f_u^0 \cos \alpha t$ (u is the last supplier)

↓

$$\delta\lambda_u(t) = f_u^0 F \cos(\alpha t - \varphi)$$

where $$\tan\varphi = -\frac{2\gamma\alpha}{\alpha^2 - {w_0}^2} \qquad F = \frac{1}{\sqrt{(\alpha^2 - {w_0}^2)^2 + 4\gamma^2\alpha^2}}$$

Dynamical solution in the vicinity of the stationary state (III)

$$f_b(t) = \frac{|W'(N_0)|}{\tau}\left(\delta\lambda_{b+1}(t) + \Delta t \frac{d\delta\lambda_{b+1}(t)}{dt}\right)$$

For b = u-1

$$f_{u-1}(t) = \frac{|W'(N_0)|}{\tau}\left(\delta\lambda_u(t) + \Delta t \frac{d\delta\lambda_u(t)}{dt}\right)$$

$$\delta\lambda_u(t) = f_u^0 F \cos(\alpha t - \varphi)$$

$$f_{u-1}(t) = f_{u-1}^0 \cos(\alpha t - \varphi - \delta_{u-1})$$

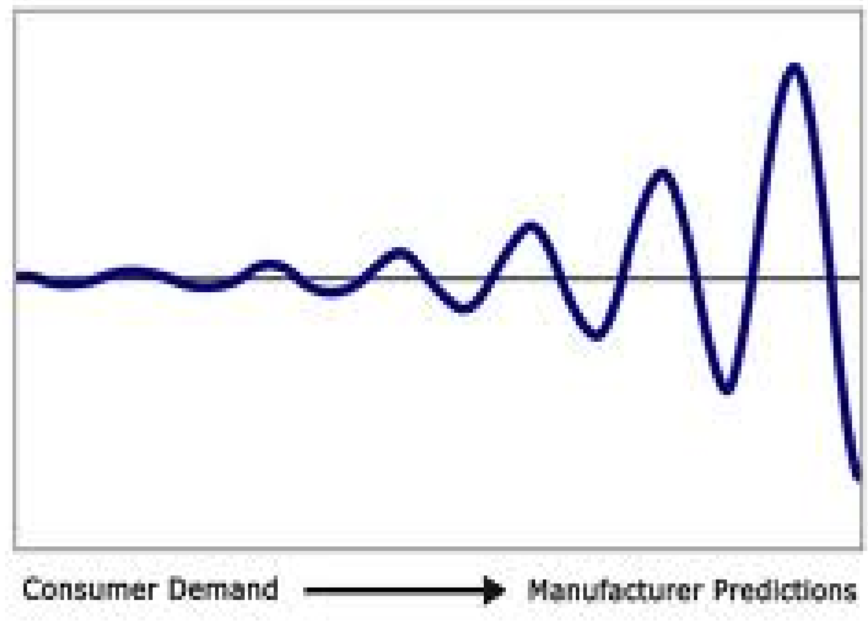
Bullwhip effect

$$f_{u-1}(t) = f_{u-1}^{0}\cos(\alpha t - \varphi - \delta_{u-1})$$

$$f_{u}(t) = f_{u}^{0}\cos\alpha t$$

$$f_{u-1}^{0} = \frac{|W'(N_0)|}{\tau} f_{u}^{0} F\sqrt{1+(\alpha\Delta t)^2}$$

(With $\tan\delta_{u-1} = -\alpha\Delta t$)



If $\frac{f_{u-1}^{0}}{f_{u}^{0}} > 1$ OSCILLATIONS INVENTORIES WILL INCREASE

Optimal velocity model

-Dynamics distance between vehicles:

$$\frac{dd_b(t)}{dt} = -\left(v_b(t) - v_{b+1}(t)\right) \iff \frac{dN_b(t)}{dt} = \lambda_b(t) - \lambda_{b+1}(t)$$

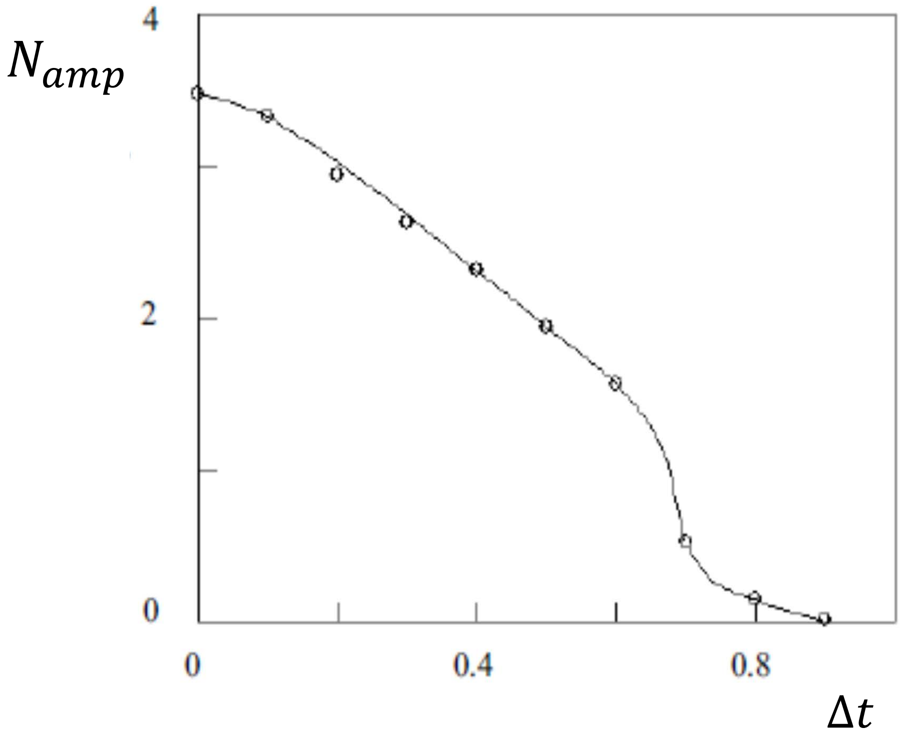
-Dynamics velocity vehicles:

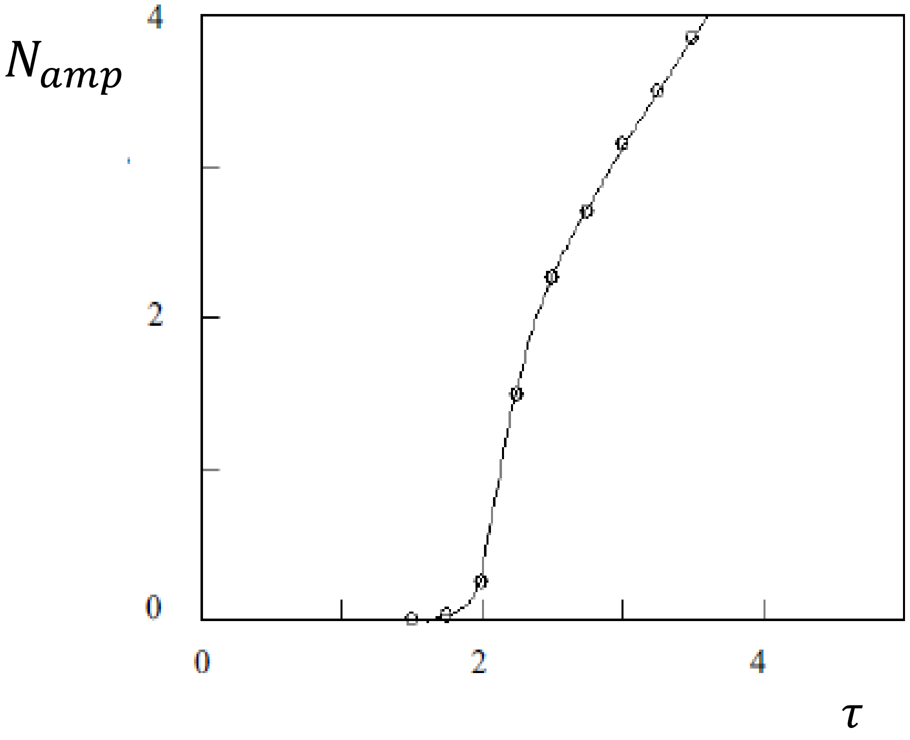
$$\frac{dv_b(t)}{dt} = \frac{V_{opt}(d_b(t)) - v_b(t)}{\tau} \iff \frac{d\lambda_b(t)}{dt} = \frac{\left(W_b(t) - \lambda_b(t)\right)}{\tau}$$

The choice of V_{opt} corresponds to the choice of the control strategy

Control strategies (I)

- Effects due to choice of adaptation time τ and forecast time horizon Δt (considering the stability condition).



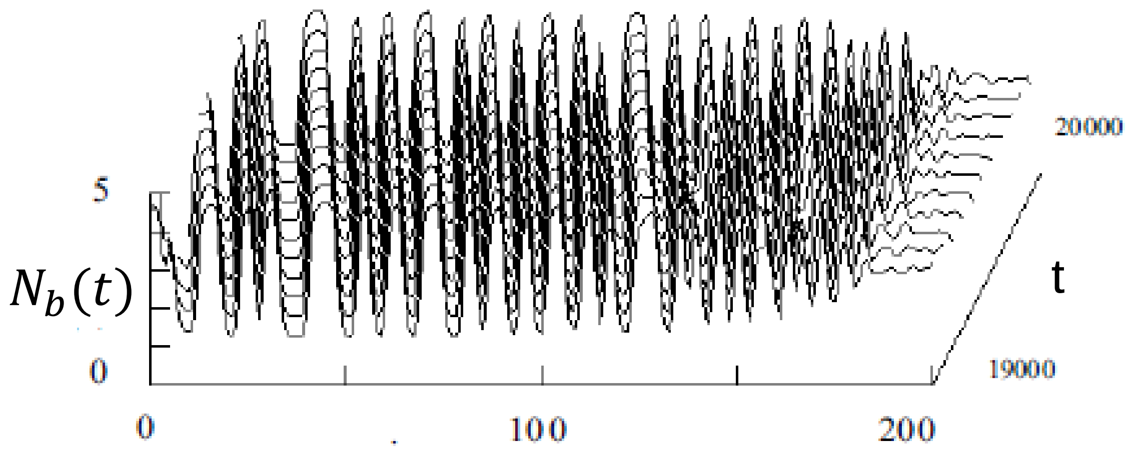


Control strategies (II)

- Adaptation of the consumption rate to either downstream suppliers (pull strategies) or upstream suppliers (push strategies).

Push strategy

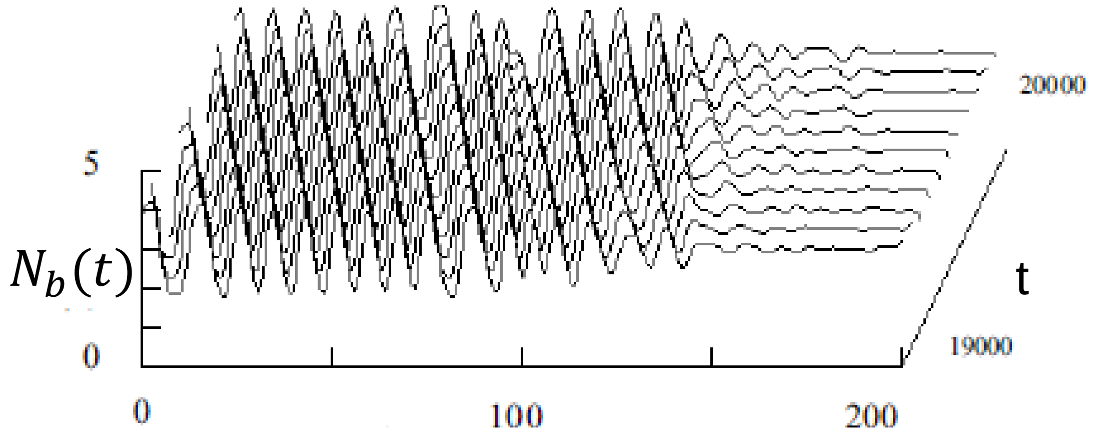
$\Delta t = 0$



Supplier b

Pull strategy

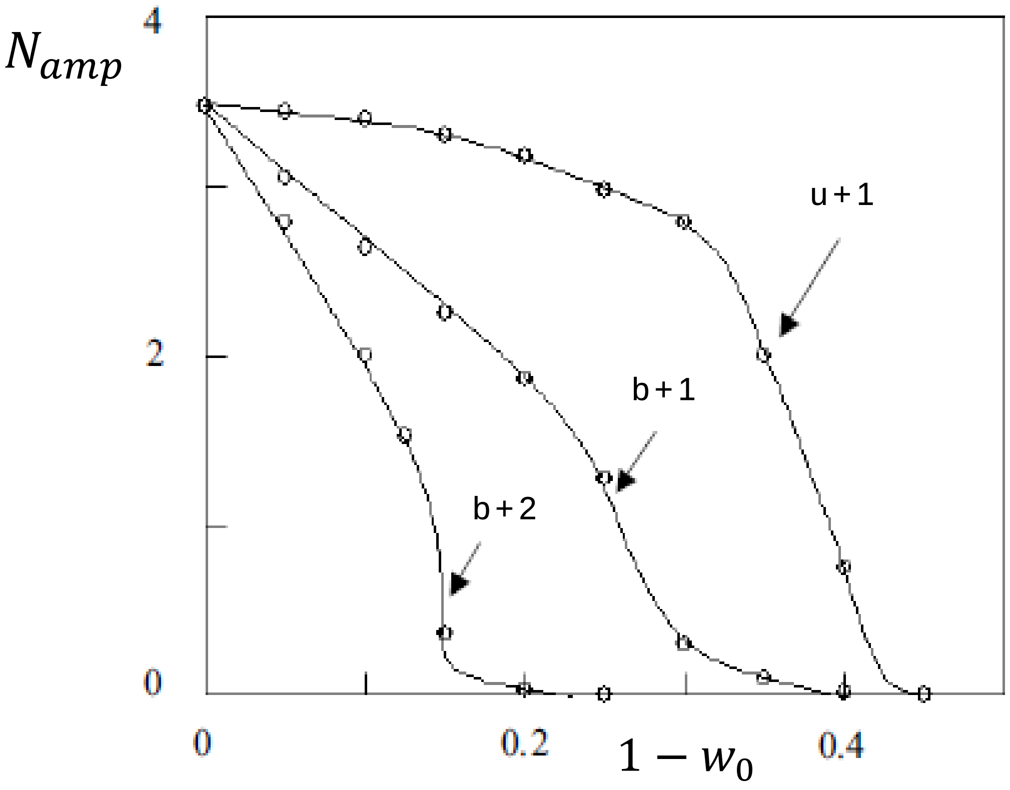
$\Delta t = 0.4$



Supplier b

Control strategies (III)

- Taking in account the inventories of other suppliers N_a by choosing specific weights in the management function.



$$N_{(b)}(t) = w_0 N_b(t) + (1 - w_0) N_a(t)$$

With a = b+1, b+2, u+1

Conclusions

It should be possible to obtain the stabilizing effects due to the choice of a specific control strategy in more complex systems than the one-dimensional supply chain

- Limited buffers
- Limited feeding rates
- Supply networks

Thanks for your attention!